Chapter 4
Hydraulic Computations and Modeling of Ice-Covered Rivers

4-1. Introduction

This chapter describes the general concepts for numerical modeling of the hydraulics of ice-covered channels and contains background material and the equations used. The calculation of the hydraulics of rivers for open water conditions (i.e., water-surface profiles) has a long history and well established procedures. One of the complications imposed by ice on rivers is the difficulty of calculating the hydraulic parameters of interest when the flow is affected by an ice cover or an ice jam. Section I of this chapter presents the general principles and equations for modeling river ice covers. Over 30 years ago, the Corps of Engineers' Hydrologic Engineering Center (HEC) formulated the first version of the program known as HEC-2 for calculating the hydraulics of open-channel flow (U.S. Army 1990). In an effort to model the effect of an ice cover, a utility program called ICETHK was developed at CRREL to be used in conjunction with HEC-2. Section II of this chapter describes the ICETHK model. More recently, the HEC-RAS model (for River Analysis System) was developed by the Hydrologic Engineering Center as a replacement for HEC-2. HEC and CRREL collaborated to include river ice as an integral part of the structure of the new model. As such, HEC-RAS overcomes several limitations that exist in ICETHK, and it applies to a wider variety of river ice situations. The ice-handling characteristics of HEC-RAS are described in Section III.

a. ICETHK. ICETHK is a useful engineering tool, since many flood studies and hydraulic design projects require the calculation of ice-affected stages. Before the development of ICETHK, the calculation of ice-affected backwater profiles using HEC-2 was painstaking, requiring many iterations. The model has two strong points. First, ICETHK is used in conjunction with HEC-2, the most commonly used backwater model in the United States, and river geometry data in the HEC-2 format are widely available. Second, ICETHK is designed to help the user understand ice jam processes and is relatively easy to use. The original ICETHK model has been supplanted by an improved ice routine in HEC-RAS, but it is described in this chapter for those who may continue to find it useful and because of its strong association with the well-established HEC-2 model.

b. HEC-RAS. The HEC-RAS model of river hydraulics contains code that enables the user to model ice-covered channels at two levels. The first level applies to an ice cover with known geometry. In this case, the user specifies the ice cover thickness and roughness at each cross section. Different ice cover thicknesses and roughnesses can be specified for the main channel and for each overbank, and both the thickness and roughness can vary along the channel. The second level addresses a wide-river ice jam. In this case, the ice thickness is determined by an ice jam force balance. The ice jam can be confined to the main channel or can include both the main channel and the overbanks. The material properties of the wide-river jam can be selected by the user and can vary from cross section to cross section. The user can specify the hydraulic roughness of the ice jam, or HEC-RAS will estimate the hydraulic roughness on the basis of empirical data.

Section I
Modeling River Ice Covers

4-2. General

The common formation of ice covers on rivers during the cold winter months arises in a variety of ways. How an ice cover forms depends on the channel flow conditions and the amount and type of ice generated. In most cases, river ice covers float in hydrostatic equilibrium because they react both elastically and plastically (the plastic response being termed *creep*) to changes in water level. The thickness and roughness of ice covers can vary significantly along the channel and even across the channel. A stationary, floating ice cover creates an additional fixed boundary with an associated hydraulic roughness. An ice cover also makes a portion of the channel cross-sectional area unavailable for flow, i.e., that part occupied by the ice. The net result is generally to reduce the channel conveyance, largely by increasing the wetted perimeter and reducing the hydraulic radius of a channel, but also by modifying the effective channel roughness and reducing the channel flow area.

4-3. Modeling Ice Covers with Known Geometry

The conveyance of a channel or any subdivision of an ice-covered channel, K_i, can be estimated using the Manning equation:

$$K_i = \frac{1.486}{n_c} A_i R_i^{2/3} \tag{4-1}$$

where

n_c = composite roughness

A_i = flow area beneath the ice cover

R_i = hydraulic radius modified to account for the presence of ice.

The composite roughness of an ice-covered river channel can be estimated using the Belokon-Sabaneev equation as

$$n_c = \left(\frac{n_b^{3/2} + n_i^{3/2}}{2} \right)^{2/3} \tag{4-2}$$

where

n_b = roughness value for the bed

n_i = roughness value for the ice.

The hydraulic radius of an ice-covered channel is found as

$$R_i = \frac{A_i}{P_b + B_i} \tag{4-3}$$

where

P_b = wetted perimeter associated with the channel bottom and sideslopes

B_i = width of the underside of the ice cover.

It is interesting to estimate the influence that an ice cover can have on the channel conveyance. For example, if a channel is roughly rectangular in shape and much wider than it is deep, then its hydraulic radius will be approximately cut in half by the presence of an ice cover. Assuming that the flow area remains constant, we see that the addition of an ice cover, having a roughness equivalent to the bed roughness, reduces conveyance by 37 percent.

4-4. Modeling Wide-River Ice Jams

The wide-river ice jam is probably the most common type of river ice jam (Figure 4-1). In this type, all stresses acting on the jam are ultimately transmitted to the channel banks. The stresses are estimated using the ice-jam force balance equation:

$$\frac{d\ \bar{\sigma}_x t}{dx} + \frac{2\tau_b t}{B} = \rho' g S_w + \tau_i \tag{4-4}$$

where

$\bar{\sigma}_x$ = longitudinal stress (along stream direction)

t = the accumulation thickness

τ_b = shear resistance of the banks

B = accumulation width

ρ' = ice density

g = acceleration of gravity

S_w = water surface slope

τ_i = shear stress applied to the underside of the ice by the flowing water.

This equation balances changes in the longitudinal stress in the ice cover and the stress acting on the banks with the two external forces acting on the jam, namely the gravitational force attributable to the slope of the water surface and the shear stress of the flowing water on the jam underside.

a. Assumptions. Two assumptions are implicit in this force balance equation: that $\bar{\sigma}_x$, t, and τ_i are constant across the width, and that none of the longitudinal stress is transferred to the channel banks

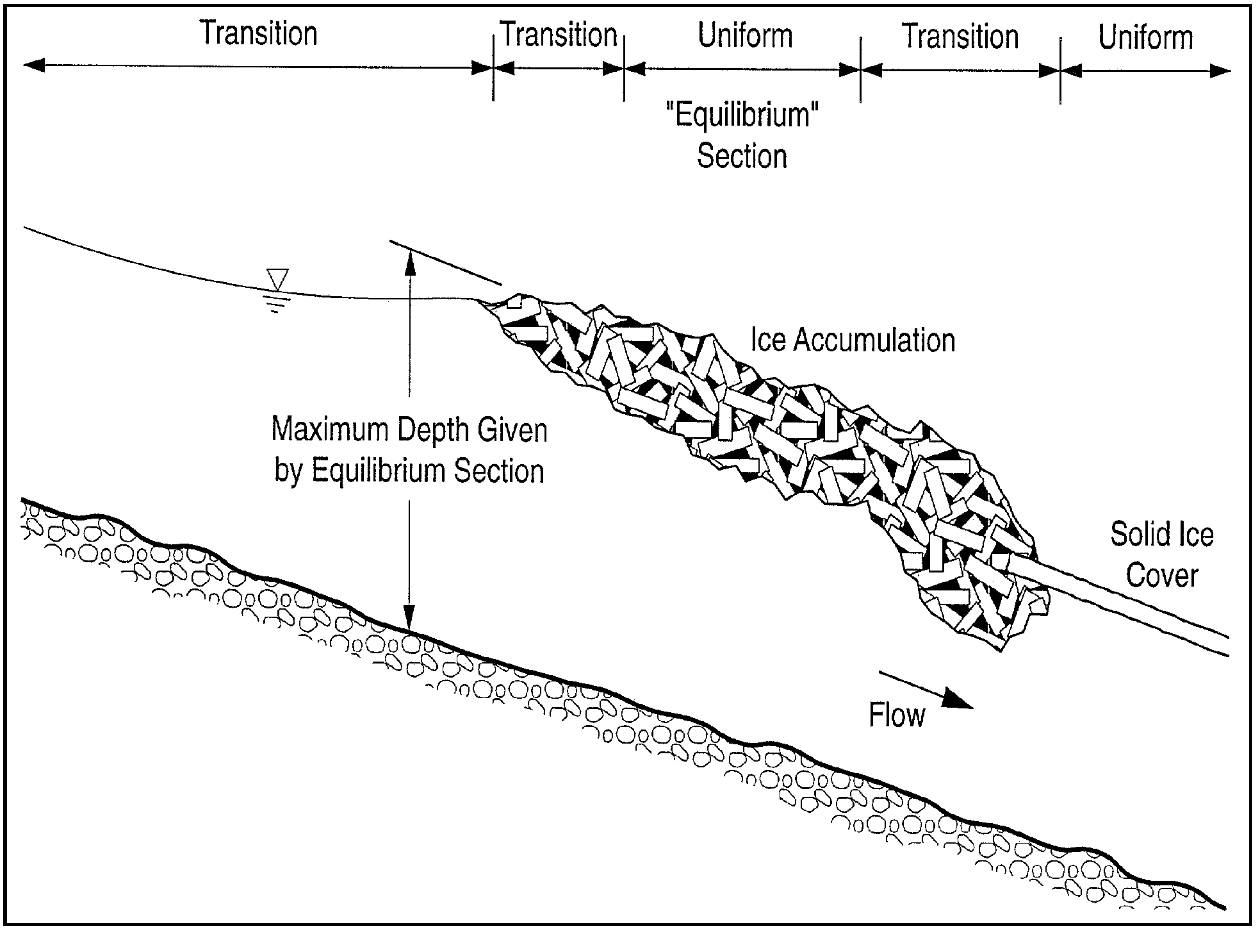

Figure 4-1. Schematic profile of a wide-river ice jam. *Note that the ICETHK model applies to the "equilibrium section" of the jam where ice thickness and flow are relatively uniform. HEC-RAS applies to the entire jam except grounded portions, if any*

through changes in stream width or horizontal bends in the plan form of the river. In addition, the stresses acting on the jam can be related to the mean vertical stress using the passive pressure concept from soil mechanics, and the mean vertical stress results only from the hydrostatic forces acting in the vertical direction. In the present case, we also assume that there is no cohesion between individual pieces of ice, a reasonable assumption for ice jams formed during river ice breakup.

(1) In this light, the vertical stress, $\bar{\sigma}_z$, is

$$\bar{\sigma}_z = \gamma_e t \tag{4-5a}$$

in which

$$\gamma_e = \frac{1}{2} \rho' g \ (1 - s) \ (1 - e) \tag{4-5b}$$

where

e = ice jam porosity (assumed to be the same above and below the water surface)

s = specific gravity of ice.

(2) The longitudinal stress is then

$$\bar{\sigma}_x = k_x \, \bar{\sigma}_z \tag{4-6}$$

where

$k_x = \tan^2 (45° + \phi/2)$

ϕ = angle of internal friction of the ice jam.

(3) The lateral stress perpendicular to the banks can also be related to the longitudinal stress as

$$\bar{\sigma}_y = k_1 \, \bar{\sigma}_x \tag{4-7}$$

where

k_1 = coefficient of lateral thrust.

(4) Finally, the shear stress acting on the bank can be related to the lateral stress

$$\tau_b = k_o \, \bar{\sigma}_y \tag{4-8}$$

where

$k_o = \tan \phi$.

b. Reformulation of the force balance equation. Using the above expressions, we can restate the ice-jam force balance as

$$\frac{dt}{dx} = \frac{1}{2 \, k_x \, \gamma_e} \left(\rho' g \, S_w + \frac{\tau_i}{t} \right) \frac{k_o k_1 \, t}{B} = F \tag{4-9}$$

where

F = shorthand description of the force balance equation.

4-5. Roughness of the Ice Accumulation

Ice roughness can be calculated as a function of ice thickness or as a function of ice piece size. Existing field data show that thick jams are typically made up of larger ice pieces and are hydraulically rougher than thin jams. Relationships based on Nezhikhovskiy's (1964) data relate Manning's *n* for the ice cover to the ice accumulation thickness. The relationships take the form of a similar equation by Beltaos (1983). Nezhikhovskiy's data were measured in wide canals, 2–3 meters (6.6–9.8 feet deep), for ice floes, dense slush, and loose slush.

a. Thick breakup jams. For breakup ice jams with ice accumulations greater than 0.46 meters (1.5 feet) thick:

$$n_i = 0.0588 \left(\frac{H}{2} \right)^{-0.23} t_i^{0.40} = 0.0690 \; H^{-0.23} \, t_i^{0.40} \tag{4-10}$$

where

H = total water depth

t_i = measured thickness of the ice accumulation.

b. Thin breakup jams. A second relationship for breakup ice jams applies to ice accumulations less than 0.46 meters (1.5 feet) thick:

$$n_i = 0.0506 \left(\frac{H}{2} \right)^{-0.23} t_i^{0.77} = 0.0593 \; H^{-0.23} \, t_i^{0.77} \tag{4-11}$$

c. Freezeup jams. A third relationship predicts the roughness of a freezeup ice jam:

$$n_i = 0.0249 \left(\frac{H}{2} \right)^{-0.23} t_i^{0.54} = 0.0292 \; H^{-0.23} \, t_i^{0.54} \tag{4-12}$$

d. Roughness summary. Nezhikhovskiy's data and the curves produced by these three equations are plotted in Figure 4-2.

4-6. Limitations of Ice Modeling

Although there are a number of limitations that arise from the assumptions required to solve the ice-jam force balance in practical situations, the models have produced good results in a number of applications. There are two general classes of limitations: those associated with the circumstances of the jam formation, and those describing the material properties of the jam.

a. Limitations attributable to circumstances of jam formation. Both HEC-RAS and ICETHK assume one-dimensional, gradually varied, steady flow. This may be in error when the ice jam formed during a surge or other transient flow event. However, the extent to which the ice jam is influenced by

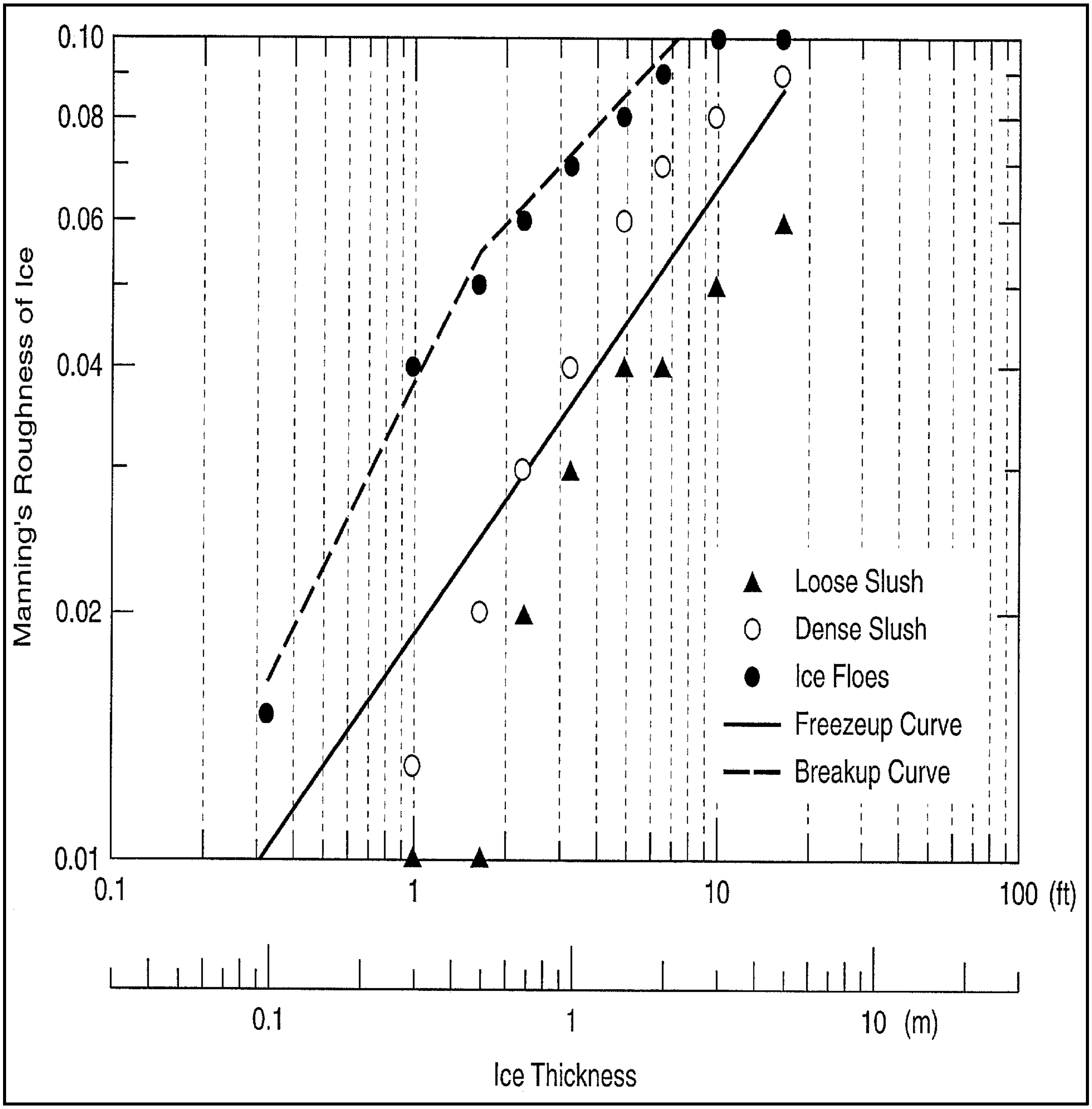

Figure 4-2. Nezhikovskiy's ice roughness values. *The data are plotted in log-log format with the ice-thickness versus ice-roughness relationships used in the ICETHK model*

the unsteady flow cannot be estimated at this time. Neither HEC-RAS nor ICETHK can estimate where an ice jam will occur. This information must be entered by the user.

b. Limitations attributable to jam material properties description. The collection of ice floes that make up the jam are assumed to be a granular material with known properties. The determination of these properties requires that the ice jam be floating in hydrostatic equilibrium. The result is that grounded ice jams, where the ice jam is resting fully or partially on the channel bottom, cannot be well described by this approach. This may have the largest influence at the downstream end or "toe" of the jam in the calculated results. However, it has generally been found that this description produces "reasonable" results in the toe area.

Section II
The ICETHK Model

4-7. General

ICETHK is an ice utility program that is used in conjunction the HEC-2 backwater model to simulate an equilibrium ice jam profile (Tuthill et al. 1998). ICETHK uses the results of hydraulic calculations from HEC-2, with an ice cover, to produce new estimates of ice thickness and ice roughness for the reach of river being modeled. HEC-2 is then used to recalculate the hydraulic conditions with the updated ice values from the previous ICETHK run. The HEC-2 and ICETHK iteration cycles continue until the change in ice thickness between successive iterations is acceptably small.

4-8. Ice Covers with Known Geometry

The utility program ICETHK cannot be used to model ice covers with known geometry (i.e., the ice cover thickness and roughness are known at every cross section). If the ice cover geometry is known, this information can be entered into HEC-2 directly using the IC card. The reader is referred to the HEC-2 Manual (U.S. Army 1990) for this information.

4-9. Equilibrium Ice Jam Theory and ICETHK

a. Definition of an equilibrium ice jam. ICETHK treats each reach between adjacent cross sections as individual equilibrium reaches. The equilibrium form of Equation 4-9 above can be found by setting the differential term with respect to *x*, the longitudinal distance, to zero. Equilibrium ice jam theory assumes that the downstream forces on the ice cover are resisted by the accumulation's internal strength and bank shear. In this case it is assumed that the downstream forces are the water drag on the ice accumulation's underside and the downstream component of the ice accumulation's weight. The ice accumulation's ability to transfer these downstream forces to the banks depends on its internal strength and thickness, and the model's governing equations determine the minimum ice thickness at which this force balance can occur.

b. Ice thickness calculation. ICETHK calculates ice thickness by three processes: juxtaposition, wide-river jam, and thinning by erosion. In this manual, only the wide-river ice jam will be discussed. In this case the wide-river jam can be simplified to a quadratic algebraic equation to reflect the ice jam forces in an equilibrium reach.

$$\mu \left(1 - \frac{\rho'}{\rho}\right) \rho' g t^2 - (g \rho' S_f B - 2_{C_i}) t - \tau_i B = 0 \qquad (4\text{-}13)$$

where

μ = coefficient related to the internal strength of the accumulation, ranging from 0.8 to 1.3

ρ, ρ' = densities of water and ice, respectively

g = acceleration due to gravity

t = thickness of the ice accumulation

S_f = friction slope (assumed equal to the water surface slope)

B = channel width at bottom of ice cover

C_i = cohesion factor for ice can range from zero for breakup jams to 958 Pa (20 lb/ft^2) for freezeup jams

τ_i = shear force on underside of accumulation, approximated by $\rho\ g(y_i/2)\ S_f$, where y_i = under-ice depth.

This quadratic equation can be solved directly.

4-10. Ice in Overbank Areas

Once flow depth in the floodplain reaches a threshold value, ice thickness in the overbank areas is determined by the same steps and equations as the channel ice thickness. The threshold floodplain depth is defined by a specified factor times the ice thickness before breakup. The use of the same calculation method to calculate ice thickness in the overbank area (i.e., the same method as is used for the main channel area) relies on the assumption that the ice-on-ice shear between the channel and floodplain ice is approximately equivalent to the bank shear of a jam remaining in the channel.

4-11. Structure and Operation of ICETHK

ICETHK is designed as a utility program for HEC-2. Figure 4-3 shows the program's overall structure and the interaction between ICETHK and HEC-2. Square-cornered boxes signify ICETHK programs and subprograms, while boxes with rounded corners indicate external input and output files. Overall, the structure is fairly simple: ICETHK reads hydraulic data from a HEC-2 output file. Then the thickness and roughness of the equilibrium ice accumulation are calculated. If water current velocity is greater than the threshold velocity for thinning, thinning of the ice accumulation is calculated, as previously described. If juxtaposition is possible, thickening from juxtaposition is found. The shoving thickness of the accumulation is then calculated, and the greater of the shoving and juxtaposition thicknesses is selected. The thickness of the initial (parent) ice cover is used as a minimum. This means that the cover cannot thin beyond the parent ice thickness. It also means that, if a solution is not possible by juxtaposition or shoving, the parent ice thickness will be used. Next, the ice roughness is calculated as a function of accumulation thickness. If floodplain flow depth is greater than a user-defined threshold value, the process described for the main channel is repeated to calculate ice thickness in the overbank areas. Finally, the resulting ice data are inserted into the original HEC-2 input file, creating a new input file.

Section III
The HEC-RAS Model

4-12. General

HEC-RAS allows the user to model ice-covered channels at two levels. The first level is an ice cover with known geometry. In this case, the user specifies the ice cover thickness and roughness at each cross

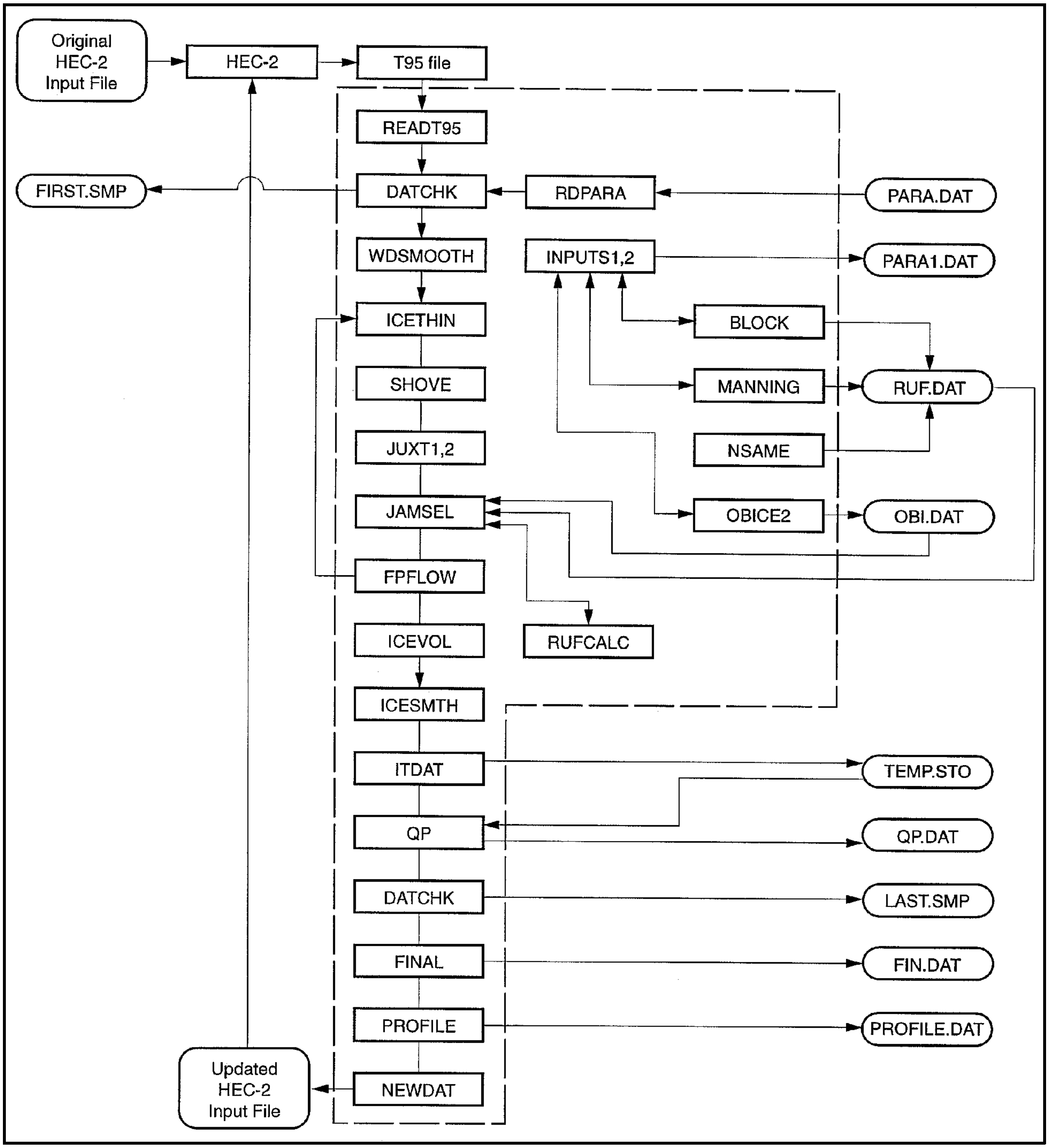

Figure 4-3. Structure of the ICETHK model. *Square-cornered boxes indicate programs and subprograms. ICETHK subprograms lie within the large dashed line box. External files (both input and output) are indicated by round-cornered boxes*

section. Different ice cover thicknesses and roughnesses can be specified for the main channel and for each overbank and both can vary along the channel. The second level is a wide-river ice jam. In this case, the ice jam thickness is determined at each section by balancing the forces on it. The ice jam can be confined to the main channel or can include both the main channel and the overbanks. The material properties of the wide-river jam can be selected by the user and can vary from cross section to cross section. The user can specify the hydraulic roughness of the ice jam or HEC-RAS will estimate the hydraulic roughness on the basis of empirical data. Published documentation (U.S. Army 1998a, 1998b, 1998c) should be consulted for a fuller discussion of HEC-RAS.

4-13. Ice Covers with Known Geometry

Separate ice thicknesses and roughnesses can be used in HEC-RAS for the main channel and each overbank, providing the ability to have three separate ice thicknesses and ice roughnesses at each cross section. The ice thickness in the main channel and each overbank can also be set to zero. The ice cover geometry can change from section to section along the channel. The suggested range of *n* values for river ice covers is listed in Table 4-1.

Table 4-1
The Suggested Range of Manning's *n* Values for a Single Layer of Ice and for Ice Jams

Single Ice Layer

Type of Ice	*Condition*	*Manning* n *Value*
Sheet ice	Smooth	0.008 to 0.012
	Rippled ice	0.01 to 0.03
	Fragmented single layer	0.015 to 0.025
Frazil ice	New, 0.3-0.9 m (1–3 ft) thick	0.01 to 0.03
	0.9–1.5 m (3–5 ft) thick	0.03 to 0.06
	Aged	0.01 to 0.02

Ice Jams

	Manning's n *Value*		
Thickness m (ft)	*Loose Frazil*	*Frozen Frazil*	*Sheet Ice*
0.1 (0.3)	--	--	0.015
0.3 (1.0)	0.01	0.013	0.04
0.5 (1.7)	0.01	0.02	0.05
0.7 (2.3)	0.02	0.03	0.06
1.0 (3.3)	0.03	0.04	0.08
1.5 (5.0)	0.03	0.06	0.09
2.0 (6.5)	0.04	0.07	0.09
3.0 (10.0)	0.05	0.08	0.10
5.0 (16.5)	0.06	0.09	--

4-14. Ice Jam Thickness Calculation

HEC-RAS estimates the ice jam thickness using Equation 4-9 above. No assumptions are made with respect to there being an equilibrium reach or not. As a result, the entire equation is solved, including the differential term with respect to *x*, the longitudinal length along the channel.

a. Force balance. To evaluate the force balance equation, the under-ice shear stress must be estimated. The under-ice shear stress is

$$\tau_i = \rho\, g\, R_{ic} S_f \tag{4-14}$$

where

R_{ic} = hydraulic radius associated with the ice cover

S_f = friction slope of the flow.

b. Hydraulic radius. The value of R_{ic} can be estimated as

$$R_{ic} = \left(\frac{n_i}{n_c} \right)^{3/2} R_i \tag{4-15}$$

c. Roughness. The hydraulic roughness of an ice jam can be estimated using the empirical relationships derived from the data of Nezhikovskiy (1964). These are the relationships described in paragraph 4-5. Note that only the relationships for breakup ice covers are available.

4-15. Solution Procedure

The ice jam force balance equation is solved using an approach analogous to the standard step method. In this, the ice thickness at each cross section is found, starting from a known ice thickness at the upstream end of the ice jam. The ice thickness at the next downstream section is assumed and the value of *F* found. The ice jam thickness at this downstream cross section, t_{ds} , is then computed as

$$t_{ds} = t_{us} + \bar{F}\, L \tag{4-16}$$

where

t_{us} = thickness at the upstream section

L = distance between sections

$\bar{F} = (F_{us} + F_{ds})/2.$

The assumed value and computed value of t_{ds} are then compared. The new assumed value of the downstream ice jam thickness is set equal to the old assumed value plus 33 percent of the difference between the assumed and computed value. This *local relaxation* is necessary to ensure that the ice jam calculations converge smoothly to a fixed value at each cross section. A maximum of 25 iterations is allowed for convergence. The above steps are repeated until the values converge to within 0.03 meters (0.1 foot) or to a user-defined tolerance.

a. Tests for reasonableness. After the ice thickness is calculated at a section, the following tests are made:

- The ice thickness cannot completely block the river cross section. At least 0.30 meters (1.0 foot) must remain between the bottom of the ice and the minimum elevation in the channel available for flow.

- The water velocity beneath the ice cover must be less than 1.5 m/s (5 ft/s) or a user-defined maximum velocity. If the flow velocity beneath the ice jam at a section is greater than this, the ice thickness is reduced to produce a flow velocity of approximately 1.5 m/s (5 ft/s) or the user-defined maximum water velocity.

- The ice jam thickness cannot be less than the thickness supplied by the user. If the calculated ice thickness is less than this value, it is set equal to the user-supplied thickness.

b. Simultaneous solution scheme. It is necessary to solve the force-balance equation and the energy equation simultaneously for the wide-river ice jam. However, difficulties arise because the energy equation is solved using the standard step method, starting from the downstream end of the channel and proceeding upstream, while the force-balance equation is solved starting from the upstream end and proceeding downstream. The energy equation can only be solved in the upstream direction because ice covers and wide-river jams exist only under conditions of subcritical flow. To overcome this incompatibility and to solve both the energy and the ice jam force-balance equations, the following solution scheme was adopted.

(1) A first guess of the ice jam thickness is provided by the user to start this scheme. The energy equation is then solved using the standard step method starting at the downstream end. Next, the ice jam force-balance equation is solved from the upstream to the downstream end of the channel. The energy equation and ice jam force-balance equation are solved alternately until the ice jam thicknesses and water surface elevations converge to fixed values at each cross section. This is *global convergence*.

(2) Global convergence occurs when the water surface elevation at any cross section changes less than 0.02 meters (0.06 feet), or a user-supplied tolerance, and the ice jam thickness at any section changes less than 0.03 meters (0.1 foot), or a user-supplied tolerance, between successive solutions of the ice jam force-balance equation. A total of 50 iterations (or a user-defined maximum number) is allowed for convergence. Between iterations of the energy equation, the ice jam thickness at each section is allowed to vary by only 25 percent of the calculated change. This *global relaxation* is necessary to ensure that the entire water surface profile converges smoothly to a final profile.

4-16. References

a. Required publications.
None.

b. Related publications.

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